Chpt3. Quantum states of atoms: Bohr model

3-1 Background

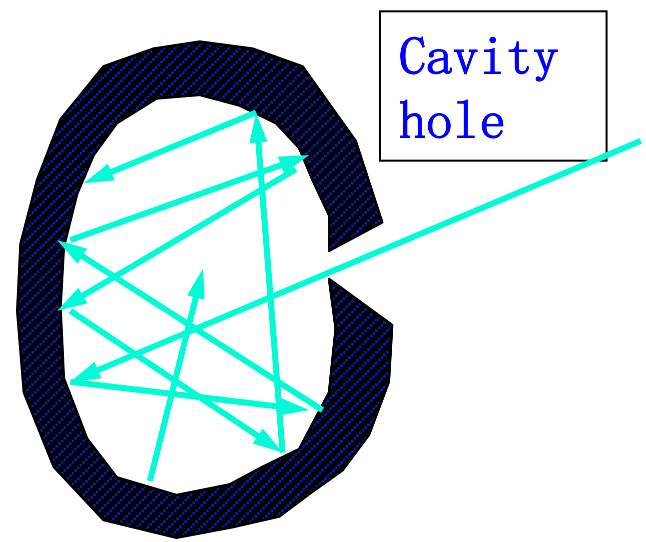
- Evidence of The quantum hypothsis I:

  Black body radiation (Max Plank 1900)
- Evidence of The quantum hypothsis II

  Photoelectric effect (Einstein 1905)
- Spectrum of light (Niels Bohr, 1913)

3.1.1 Evidence I: Black body radiation

- What is blackbody:
  - A body absorbs all light incident upon it without reflecting any light back.
  - The hole in the cavity , Sun, High T furnance
  - Not necessarily be black, no reflecting but radiation



- Thermal radiation
  - Any objects with nonzero T, radiates out EM wave
- Equilibrium thermal radiation
  - Radiation E = Absoption Energy

- Radiant intensity $R(\lambda,T)$
  - The energy emitted per unit time unit area at T around wave length λ per unit WL
- Total radiant intensity

$$R(T)=\int_0^\infty R(\lambda,T)d\lambda$$

- $R(\lambda,T)$versus $R(\nu,T)$

$$R(T)=\int_0^\infty R(\lambda,T)d\lambda=\int_0^\infty R(\nu,T)d\nu \rightarrow R(\lambda,T)d\lambda=R(\nu,T)d\nu$$

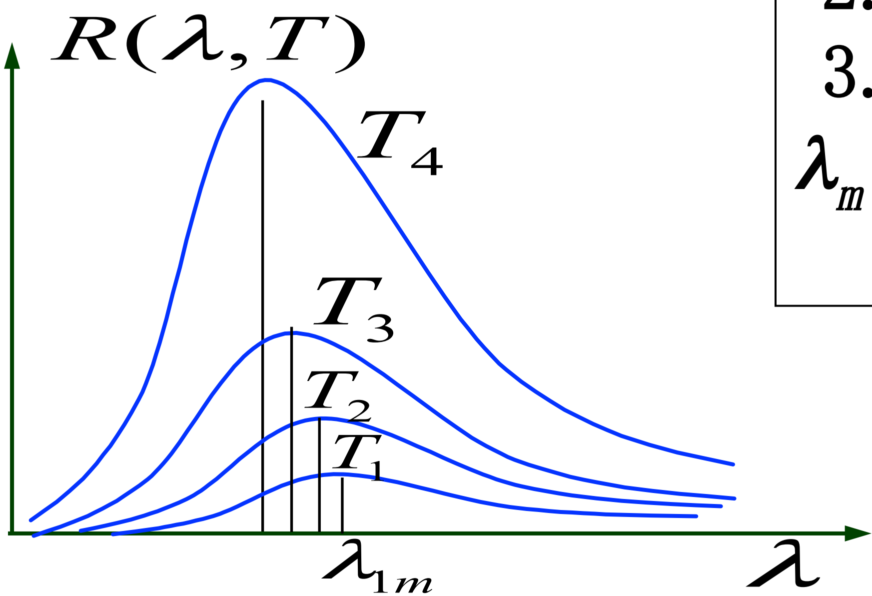
$$R(\lambda,T)=R(\nu,T)\frac{d\nu}{d\lambda}\rightarrow R(\lambda,T)=\frac{c}{\lambda^2}R(\nu,T)$$

- Energy density of radiation field $E(\nu, T)$
  - The energy density around ν per unit frequency

$$R(\nu, T) = \frac{c}{4} E(\nu, T)$$

- **Kirchhoff theorem**
  - In Eql., $E(\nu, T)$ ν curve only depens on T, independent of the cavity's structure

Radiant intensity curves



Rules:

1. increase with T
2. has a peak at λm
3. with increasing T, λ_m decreases

Theorems of BBR

- :
  - Wien theorem

$$\lambda_m T = b$$

  - Stefan-Bolzmann theorem-

$$R(T) = \sigma T^4$$

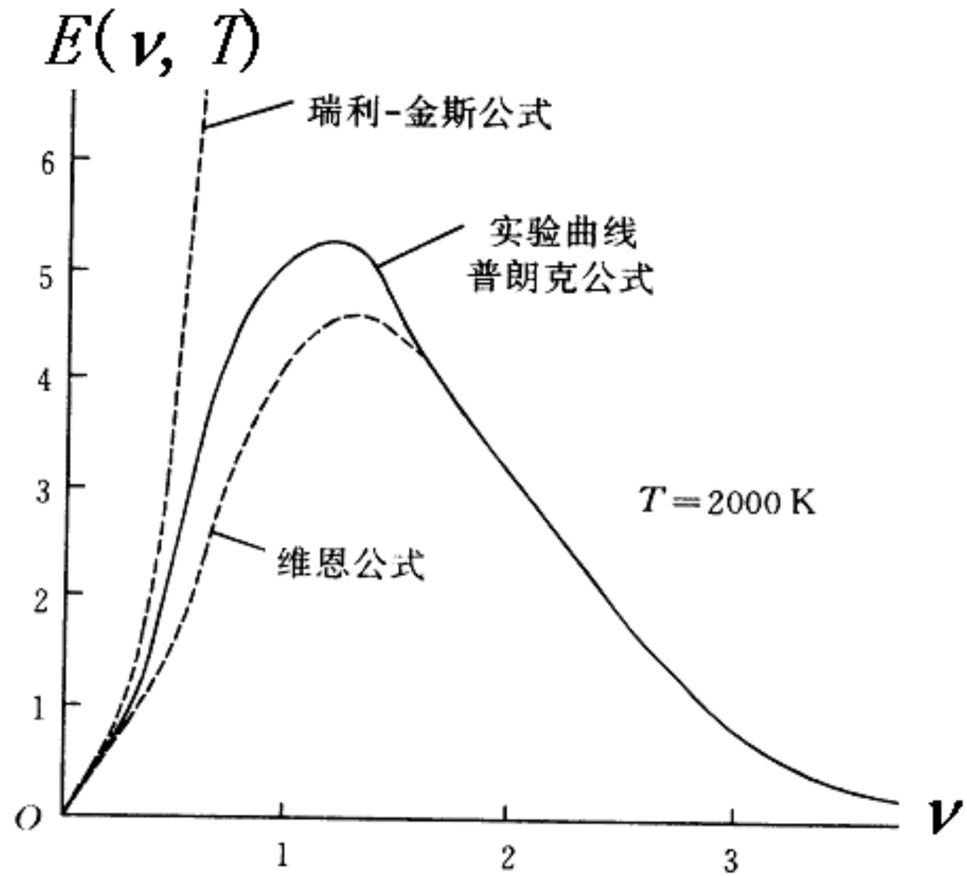
σ =5.67051×10^{-8} **W/m²·K⁴** (Stefan constant)

Classic physics runs into trouble

- Wien formula

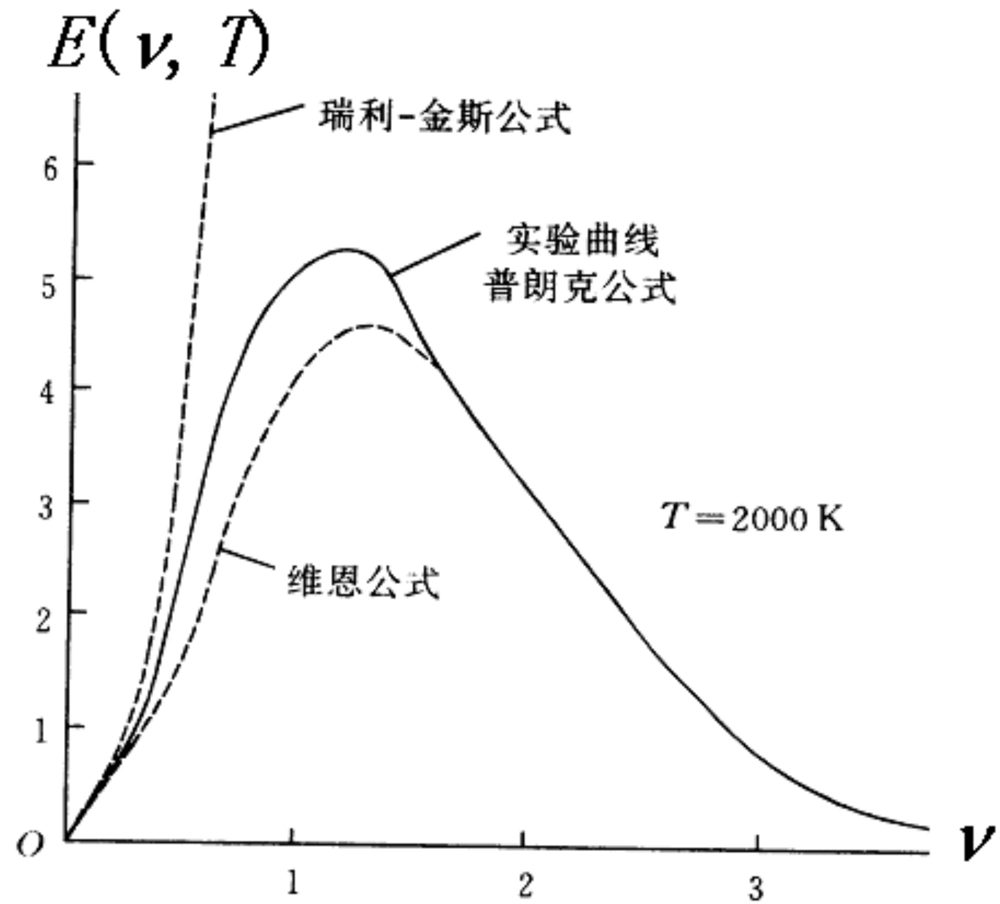
$$E(\nu,T)=C_1\nu^3e^{-\frac{C_2\mu}{T}}$$

Only works in high frequency



Raylei Jeans -- $\boxed{\text{Ultravoilet Catstrophe}}$

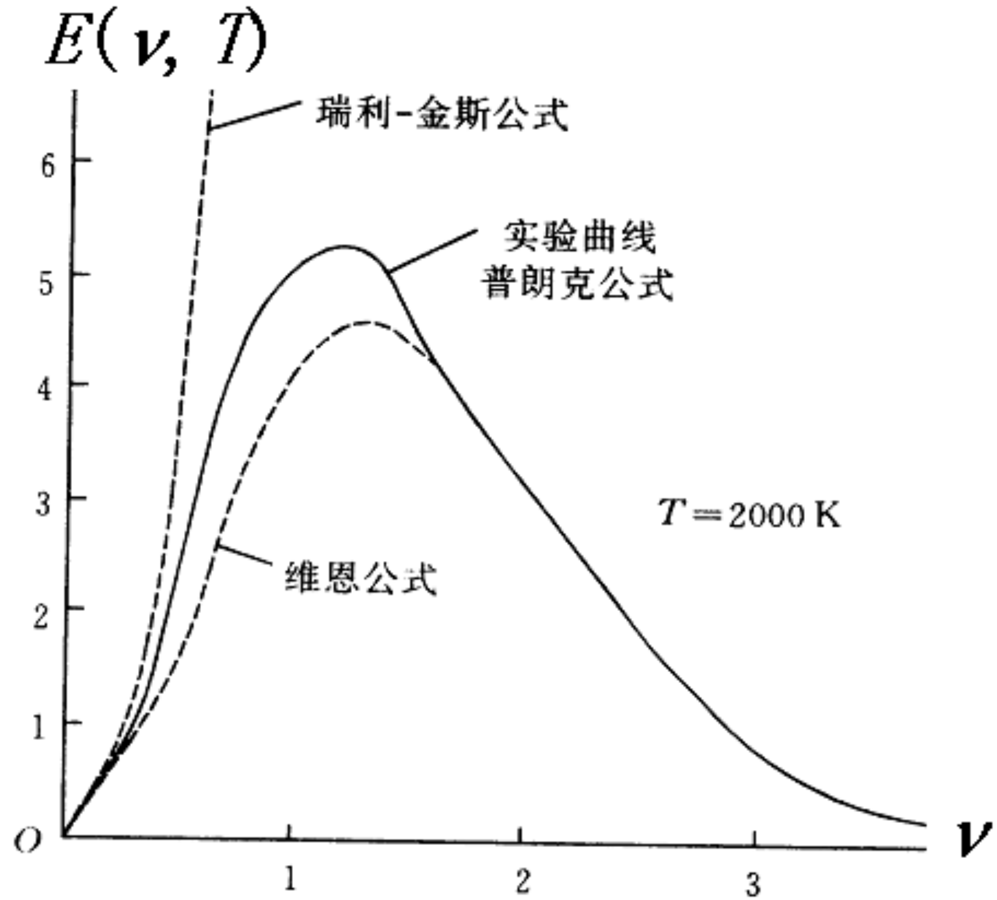
$$E(\nu,T)=\frac{8\pi\nu^2}{c^3}kT$$



Plank formula for energy distribution –inspired guess

$$E(\nu,T)=\frac{\pi h}{c^3}\frac{\nu^3}{e^{h\nu/(kT)}-1}$$

Fits experiment well



Plank's quantum hypothesis

The energy exchange of the EM radiation can only be in the form of quanta: E=n *hν*. N=1,2,3…*hν* is the energy quanta, h is the Plank's constant

$$h = 6.6260755 \times 10^{-34}\ \mathrm{J \cdot s}$$

- **Integrate the PLK formula , 0 → ∞ one obtains Stefan-Boltzmann theorem**
- Maximizing it gives Wien theorm

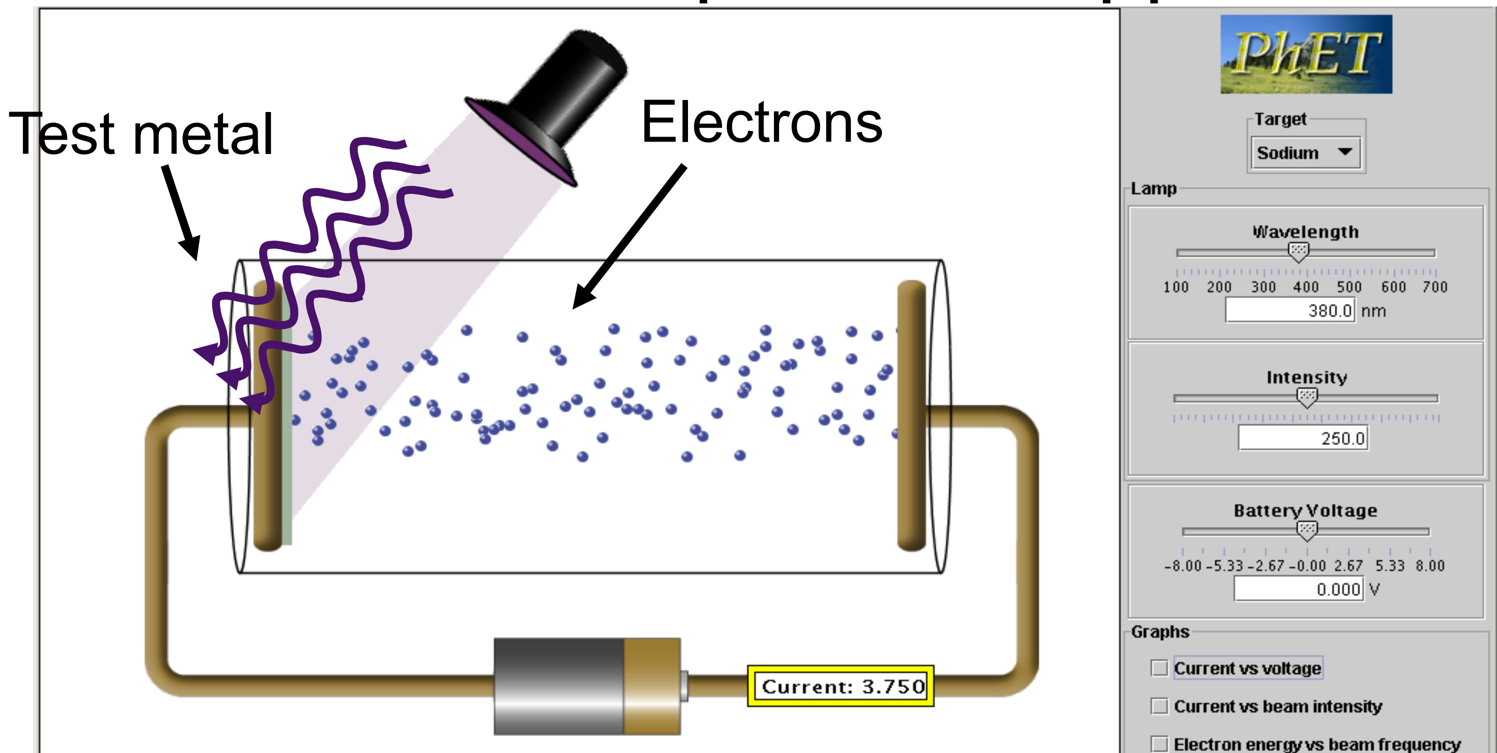
Photoelectric Effect

(How Einstein really became famous!)

3.1.2 The Photoelectric Effect

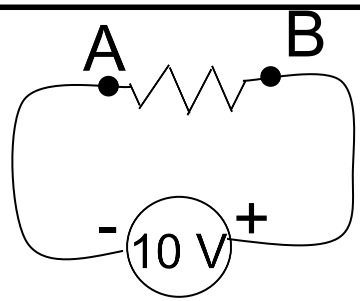
Photoelectric effect: experiment showing light is <u>also</u> a particle. Energy comes in particle-like chunks- basics of quantum physics. (energy of one chunk depends on frequency, wave-like beam of light has <u>**MANY**</u> chunks, energy of beam is sum)

Photoelectric effect experiment apparatus.



Two metal plates in vacuum, adjustable voltage between them, shine light on one plate. Measure current between plates.

I. Understanding the apparatus and experiment.

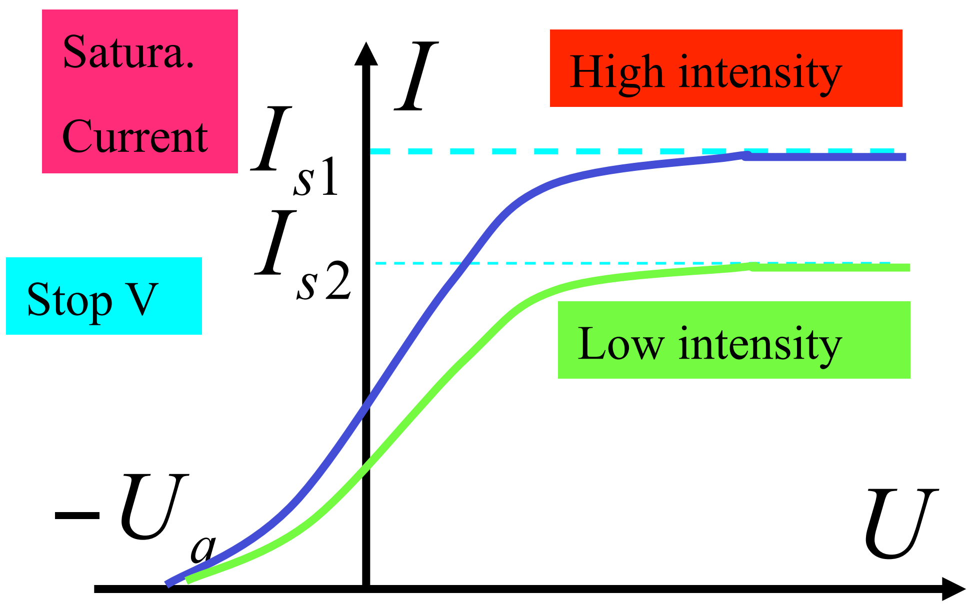


Potential difference between A and B = +10 V
Measure of energy an electron gains going from A to B.

Experimental results 1

– **PE current I is proportional to the light intensity**

emitted electron numbers proportinal to intensity



Experimental results 2

- ***With anti stopping voltage U_a*, current vanishes**

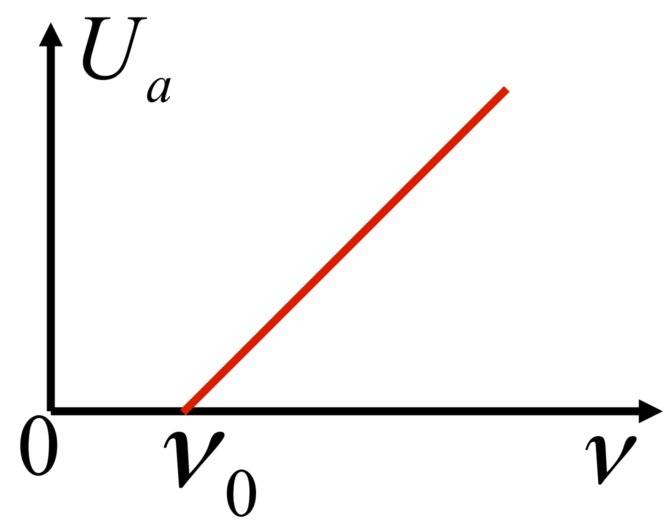
  The maximum kinetic energy

$$\frac{1}{2}mv_m^{\ 2} = eU_a$$

  – **Stoping V is linear in the freqency of incident light**

$$\frac{1}{2}mv_m^{\ 2} = e(K\nu - U_0)$$

Em is linear in \nu, Indep. of light intensity



Experimental result 3

$$\frac{1}{2}mv_m^{\ 2} = e(K\nu - U_0), \quad \frac{1}{2}mv_m^{\ 2} > 0$$

→

$$\nu \geq \frac{U_0}{K}$$ **Threshold frequency** $$\nu_0 = \frac{U_0}{K}$$

– **Current begins instantaneously(<1ns)，regardless the light intensity**

Classical explaination of PEE

- **Classical light wave theory tells**
  - **The electron kinetic energy depends on the intensity (amplitude) in contract with experiment result: dependent on frequency**

  - **For strong enough light intensity，there should be PEE for any frequency:**

    this is a threshold frenquency 存在截止频率(红限)

- **The energy gained continuously by e reached a certain value, it would be ejected from the metal**

  Experiment: **Current begins instantaneously(<1ns), regardless the light intensity**

- **Einstein's light quantum theory**
  - Photon

  Light propogates as particle flow, Photon with frequency ν has energy $h\nu$

Quantum explaination of PEE

- **Einstein's photon theory**
  - **PEE Equation**

An electron absorbs a photon ,

Photon E

$$h\nu = \frac{1}{2}mv^2 + A$$

Binding E

$$\boxed{h = eK}$$

$$\boxed{A = eU_0}$$

$$\frac{1}{2}mv_m^{\ 2} = e(K\nu - U_0)$$

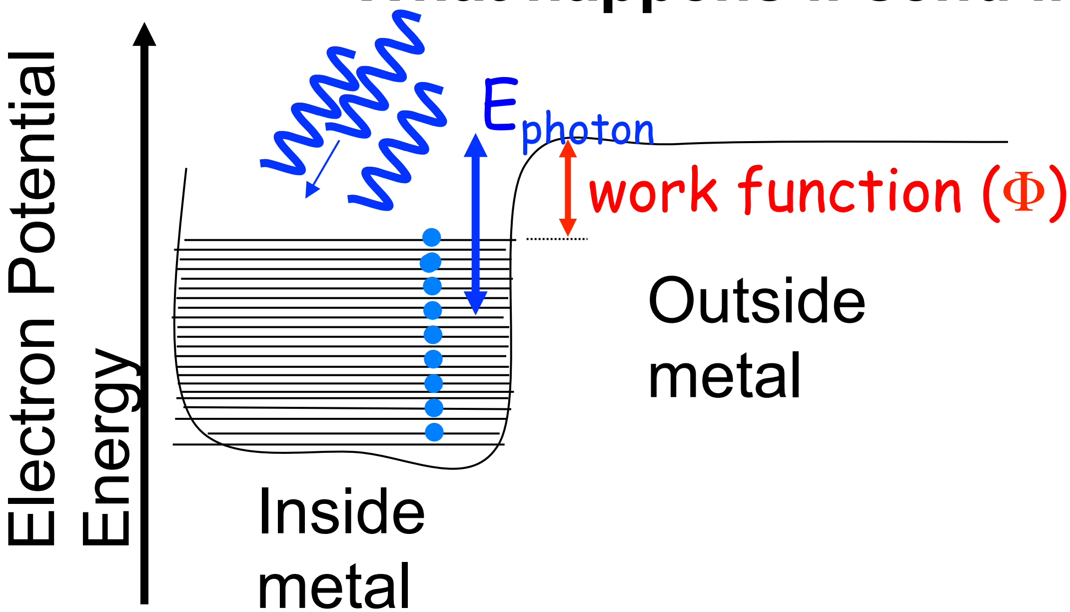
$$\boxed{\nu_0 = \frac{U_0}{K} = \frac{A}{h}}$$

Apply Conservation of Energy.

Energy in = Energy out

Energy of photon = energy needed to kick electron out of metal + Initial KE of electron as exits metal

What happens if send in bunch of blue photons?



Photon gives electron "kick of energy".

Electrons have equal chance of absorbing photon:

→ Max KE of electrons = photon energy - Φ

→ Min KE = 0

→ Some electrons, not enough energy to pop-out, energy into heat

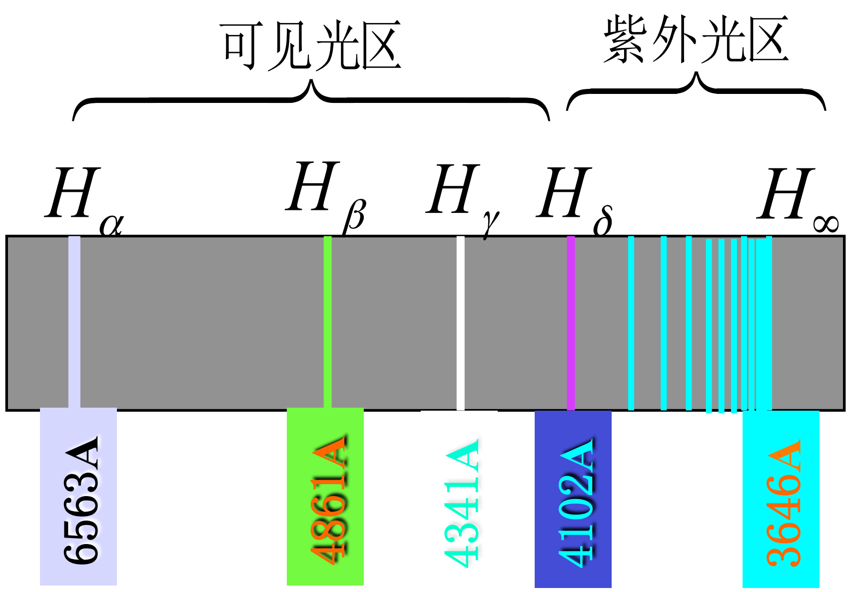
Summary of PEE by QT

- **High intensity→ more photons→ more electrons emitted→large current i**
- **Em is linear in ν , Indep. of light intensity**
- ***Only when $\nu \geq A/h$*** **, PEE happens (threshold)**
- **Light's energy discontinuous absorbed at once**

3.1.3 SPECTRUM OF LIGHT

- The intensity distribution of EM radiation of atoms as wave length ***I* (λ)**
  - Measured by spectrometer $\tilde{\nu} = \frac{1}{\lambda}$
  - Different source has its own spectrum .

hydrogen atom



Rydberg formula

$$\boxed{\tilde{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) = T(n) - T(n')} \qquad n = 1, 2, \ldots \;;$$

$R_H = 1.096776 \times 10^7 \mathrm{m}^{-1}$ **Rydberg constant**

$$T(n') = \frac{R}{n'^2}, \quad T(n) = \frac{R}{n^2} \qquad \boxed{\text{Term values}}$$

N gives the series; for a fix n, each ***n'*** gives a spectrum line

Wave number = the differnce of term values-

$$\tilde{\nu} = R_H\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

❶ **n=1, n'=2,3,…** 赖曼系， 紫外区

❷ **n=2, n'=3,4,…** 巴尔末系， 可见光区

❸ **n=3, n'=4,5,…** 帕邢系， 红外区

❹ **n=4, n'=5,6,…** 布喇开系， 红外区

❺ **n=5, n'=6,7,…** 普丰特系， 红外区

❻ **n=6, n'=7,8,…** 哈菲莱系， 红外区

3.2 Bohr model

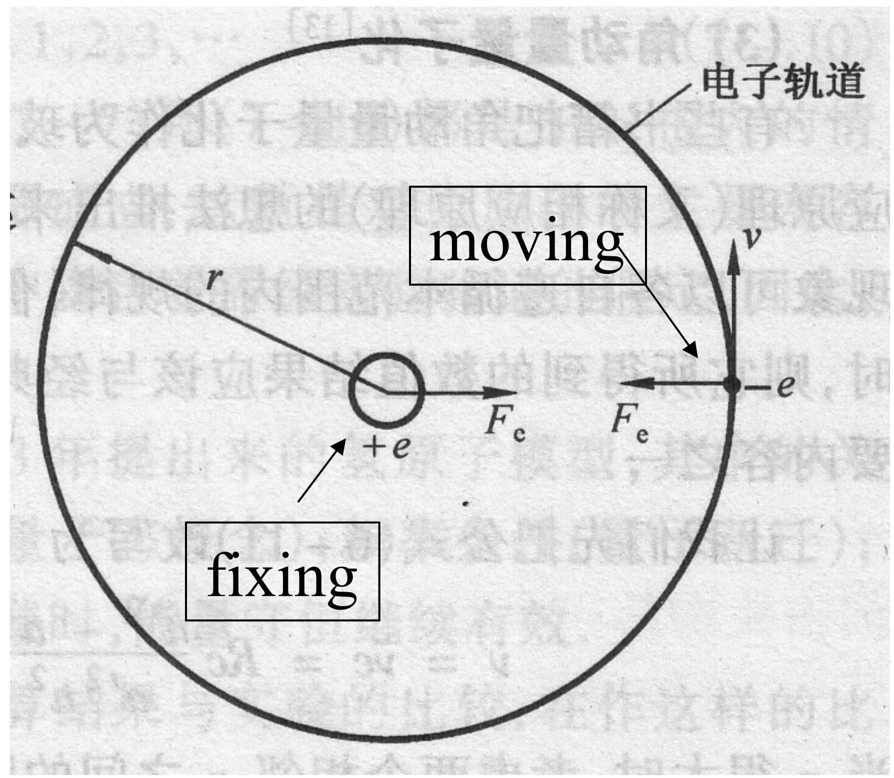
- Classic orbits with stationary state
- Frequency condition
- **Angular momentum quantization**
- **Numerical computation method**

3.2.1 Classic orbit with Stationary state

- Stationary state condition
  - Atoms can only in series stationary states with discrete energy ; electrons in certain discrete orbits, circling around without EMW radiation.

$$F = m_e \frac{v^2}{r}$$

$$m_e \frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{r^2}$$



Stationary state condition

$$E = T + V = \frac{1}{2} m_e v^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{m\frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}}$$

$$= \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r} = -T$$

– Circling moving frequency

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi r}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}} = \frac{e}{2\pi}\sqrt{\frac{1}{4\pi\varepsilon_0 m_e r^3}}$$

3.2.2 Frequency condition

- The energy emitted when an electron jumps from a state $E_{n'}$ orbit to another orbit E_n, with radiation or absorbtion of a photon energy

$$\boxed{h\nu = E_{n'} - E_n}$$

$$\frac{c}{\lambda} = c\tilde{\nu} = \nu$$

$$\tilde{\nu} = R_H\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

$$\boxed{E_n = -\frac{Rhc}{n^2}}$$

Radius quantization

$$E = -\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{r_n = \frac{e^2}{4\pi\varepsilon_0 2Rhc} n^2}$$

3.2.3 Angular momentum quantization

- **Correspondence principle**
  - **Phenomena in atomic field and in the macroscopic domain follow their own rules, but when extending the microscopic field to the classical field, numerical result should agree with that from classical rules**
- → Angular momentum quantization condition

$$\boxed{\tilde{\nu} = R\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)} \qquad \boxed{c\tilde{\nu} = \nu}$$

$$\boxed{\nu = Rc\,\frac{n'^2 - n^2}{n'^2 n^2} = Rc\,\frac{(n'+n)(n'-n)}{n'^2 n^2}}$$

Correspondence → AMQ

– $n' \uparrow$, $n' - n = 1$ →

$$\boxed{\nu = Rc\frac{(n'+n)(n'-n)}{n^2 n'^2} = Rc\frac{2n}{n^4} = \frac{2Rc}{n^3}}$$

$$\frac{2Rc}{n^3} = \frac{e}{2\pi}\sqrt{\frac{1}{4\pi\varepsilon_0 m_e r^3}}$$

CP

$$f = \frac{e}{2\pi}\sqrt{\frac{1}{4\pi\varepsilon_0 m_e r^3}}$$

$$\boxed{r = \sqrt[3]{\frac{1}{4\pi\varepsilon_0}\frac{e}{16\pi^2 R^2 c^2 m_e}}n^2}$$

$$\boxed{r_n = \frac{e^2}{4\pi\varepsilon_0 2Rhc}n^2}$$

$$\boxed{R = \frac{2\pi^2 e^4 m_e}{(4\pi\varepsilon_0)^2 ch^3}}$$

3.2.3 AMQ

$$r_n = \frac{e^2}{4\pi\varepsilon_0 2Rhc} n^2$$

$$R = \frac{2\pi^2 e^4 m_e}{(4\pi\varepsilon_0)^2 ch^3}$$

$$E_n = -\frac{Rhc}{n^2}$$

$$\boxed{r_n = \frac{4\pi\varepsilon_0 \mathrm{h}^2}{m_e e^2} n^2,}$$

$$\boxed{E_n = -\frac{m_\mathrm{e} e^4}{(4\pi\varepsilon_0)^2 2\mathrm{h}^2 n^2}}$$

$$\mathrm{h} \equiv \frac{h}{2\pi}$$

$$L = m_{\mathrm{e}} v r = m_{\mathrm{e}} \sqrt{\frac{e^2}{4\pi\varepsilon_0 m_{\mathrm{e}} r}} \cdot r = \sqrt{\frac{m_{\mathrm{e}} e^2 r}{4\pi\varepsilon_0}}$$

$$r_n = \frac{4\pi\varepsilon_0 \mathrm{h}^2}{m_e e^2} n^2 ,$$

$$\boxed{L = n\mathrm{h}, n = 1, 2, \mathrm{L}}$$

$\leftarrow$ AMQ condition

$$m\frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

– **We expect it works for any *n* , not just for large n**

3.2.4 Numerical computation

- **Composite constants**

$$\hbar c = 197\,\mathrm{fm\cdot MeV} = 197\,\mathrm{nm\cdot eV}$$

$$e^2/4\pi\varepsilon_0 = 1.44\,\mathrm{fm\cdot MeV} = 1.44\,\mathrm{nm\cdot eV}$$

$$m_e c^2 = 0.511\,\mathrm{MeV} = 511\,\mathrm{keV}$$

- 1st born orbit radius of Hydrogen

$$r_1 \equiv a_1 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{(\hbar c)^2}{m_e c^2 e^2/4\pi\varepsilon_0}$$

$$= \frac{(197)^2}{0.511\times10^6\times1.44}\,\mathrm{nm} ; \ \frac{0.039\times10^6}{0.73\times10^6}\,\mathrm{nm} ; \ 0.053\,\mathrm{nm}$$

- Energy of the electrons in H

$$E_n = -\frac{m_e e^4}{(4\pi\varepsilon_0)^2 \cdot 2\mathrm{h}^2 n^2} = -\frac{m_e c^2}{2}\left(\frac{e^2}{4\pi\varepsilon_0 \mathrm{h}c}\right)^2 \cdot \frac{1}{n^2}$$

$$\frac{e^2}{4\pi\varepsilon_0 \mathrm{h}c} \equiv \alpha \approx \frac{1}{137} \longrightarrow \boxed{E_n = -\frac{1}{2} m_e (\alpha c)^2 \frac{1}{n^2}}$$

Fine structure constant

- Energy of the ground state of H

$$E_1 = -\frac{1}{2}m_e(\alpha c)^2 = -\frac{1}{2}m_e c^2\alpha^2$$

$$= -\frac{1}{2}(0.511\times10^6)\times(\frac{1}{137})^2\mathrm{eV} \approx -13.6\mathrm{eV}$$

$$E_n = -\frac{1}{2}m_e(\alpha c)^2\frac{1}{n^2}$$

- Ionization energy
  - Define the ground state energy in H is zero , move

$$E_1 = -\frac{1}{2}m_e c^2\alpha^2 + \frac{1}{2}m_e c^2\alpha^2 = 0$$

$$E_\infty = 0 + \frac{1}{2}m_e(\alpha c)^2 = 13.6\mathrm{eV}$$

- 1st Bohr velocity

$$v_1 = \alpha c = \frac{c}{137} \qquad E_1 = -\frac{1}{2} m_e c^2 \alpha^2 = -T = -\frac{1}{2} m_e v^2$$

Relativity effect nelegiable

- **Rydberg constant**

$$R = \frac{2\pi^2 e^4 m_e}{(4\pi\varepsilon_0)^2 ch^3} = \frac{1}{2} m_e \left(\frac{e^2}{4\pi\varepsilon_0 \mathrm{hc}} \right)^2 (c)^2 \frac{1}{hc} = \frac{1}{2} m_e (\alpha c)^2 \frac{1}{hc} =$$

- Wave number

Photon energy

$$E = h\nu$$

$$\tilde{\nu} \equiv \frac{1}{\lambda} = \frac{h\nu}{hc} \qquad \lambda = \frac{hc}{E} = \frac{1.24}{E} \mathrm{nm} \cdot \mathrm{keV}$$

$$\boxed{hc = 1.24 \mathrm{nm} \cdot \mathrm{keV}}$$

3.3 Experimental evidence I :spectra

- Spectrum of Hydrogen
- Spectra of Hydrogenlike Ions
- Existance of Deuterium

3.3.1 Spectrum of Hydrogen

- **Rydberg constant**

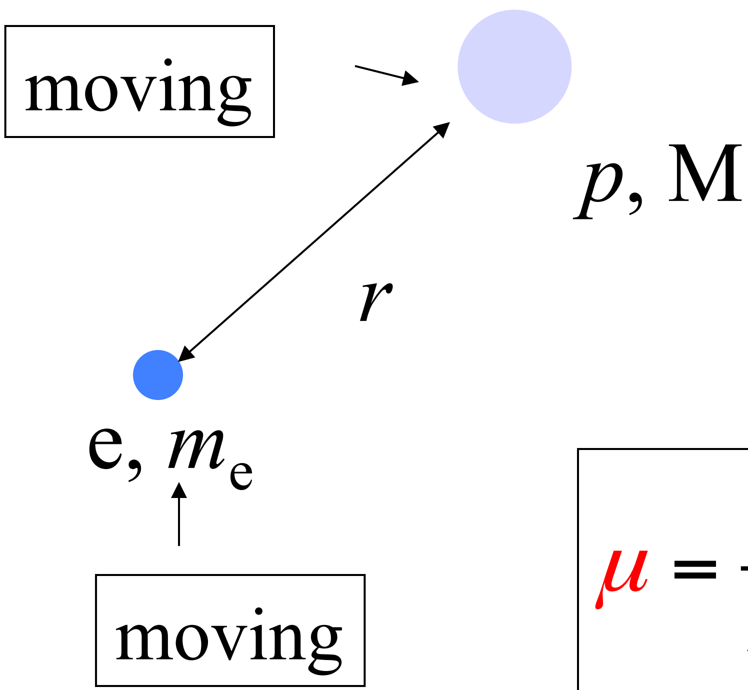
theory $$R = \frac{1}{2} m_e (\alpha c)^2 \frac{1}{hc} = 109737.315 \text{ cm}^{-1}$$

Exp. value $$R_H = 109677.58 \text{cm}^{-1}$$

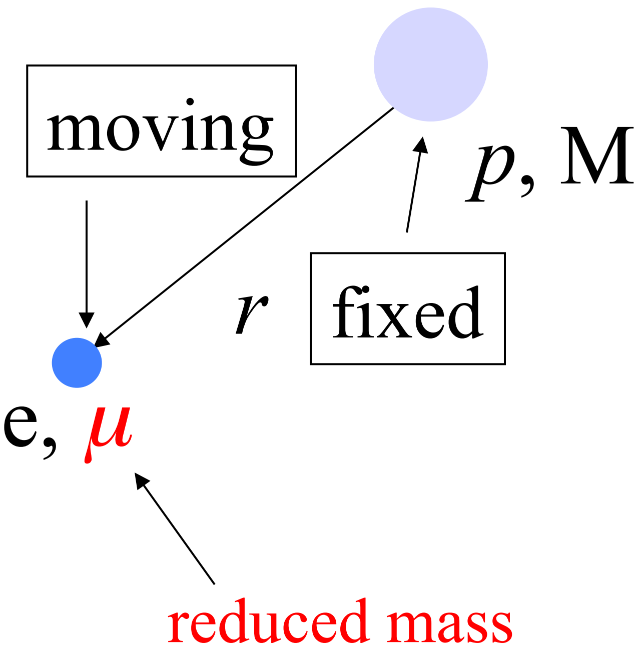
- Differenc exceeds 5e^{-4};the experimental accuracy is about 1e^{-4}:
- Stationary nucleus was taken ,
- Two-body

Nucleus –electron co-moving

Two-body problem



$\Leftrightarrow$



$$\mu = \frac{m_e}{1 + \frac{m_e}{M}}$$

- **Rydberg constant**

$$R_a = \frac{1}{2}(\alpha c)^2 \frac{1}{hc}\mu = \frac{1}{2}(\alpha c)^2 \frac{1}{hc} m_e \frac{1}{1+\frac{m_e}{M}} = R\frac{1}{1+\frac{m_e}{M}}$$

$$M \to \infty, \quad R_a = R_\infty = R$$

Exper. $\quad R_H = 109677.58\text{cm}^{-1}$

theory $\quad R_a = \frac{1}{2}(\alpha c)^2 \frac{1}{hc}\mu = R\frac{1}{1+\frac{m_e}{M}}$

$\boxed{\text{Agree}}$

Spectra of Hydrogen

- **Quantum effect**

$$\boxed{\tilde{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} = R_a\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)}$$

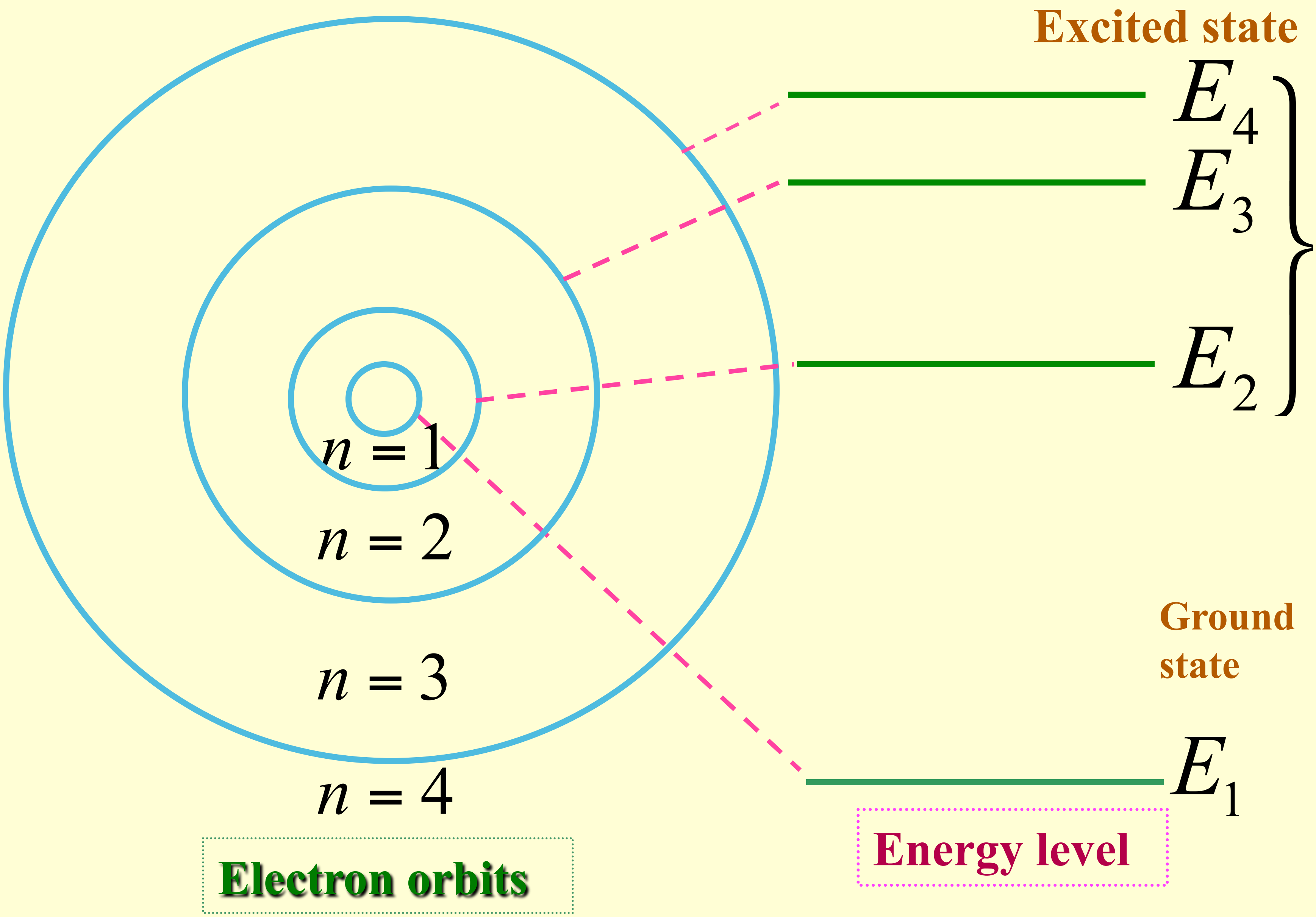
$$\boxed{E_n = -\frac{Rhc}{n^2}}$$

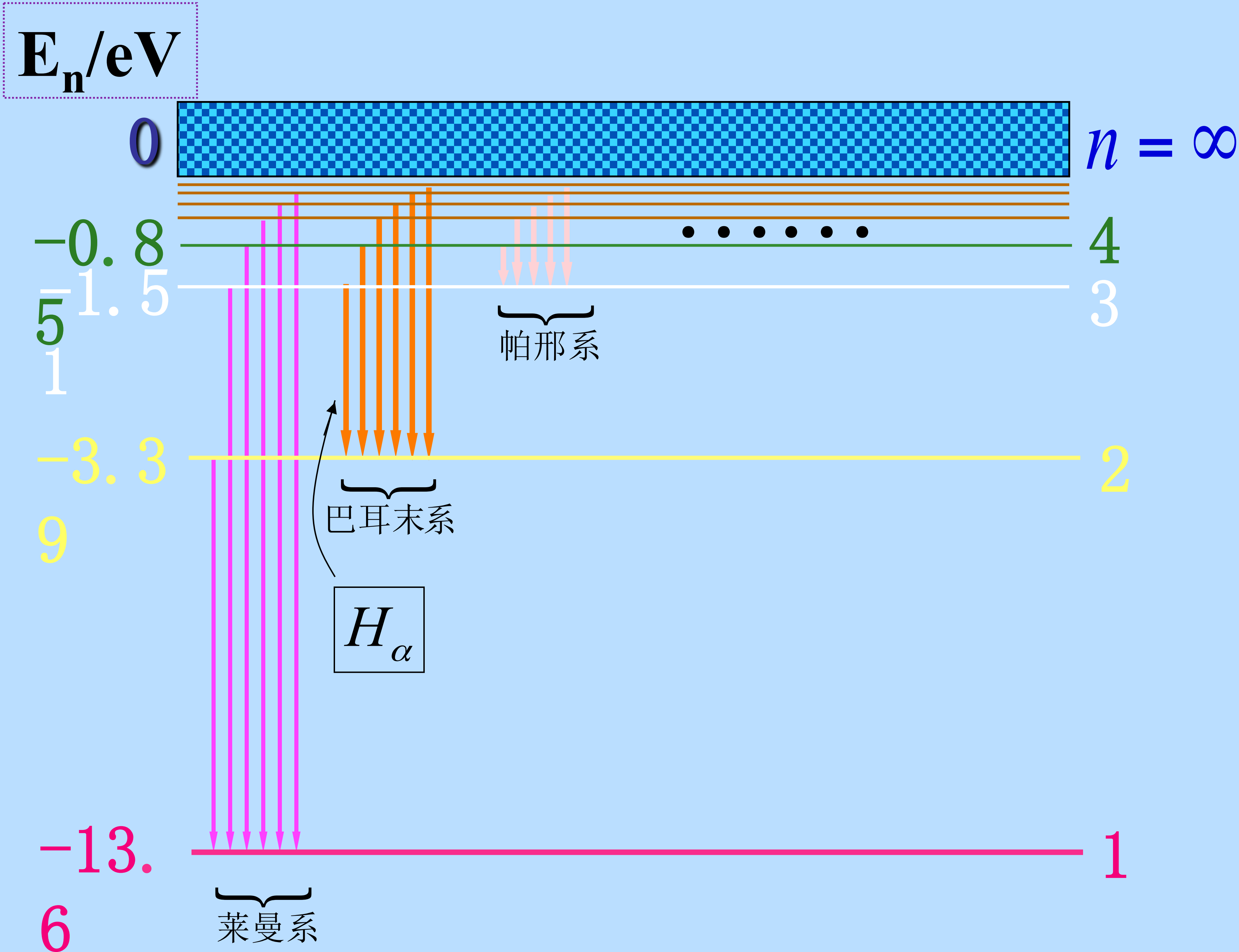
$$\boxed{h\nu = E_{n'} - E_n}$$

Final state QN

Initial state QN

Orbits & spectra lines of hydrogen atom


Excited state
E_4
E_3
E_2
$n = 1$
$n = 2$
$n = 3$
$n = 4$
Ground state
E_1
Electron orbits
Energy level



E_n/eV
0
$n = \infty$
−0.85
4
−1.51
3
帕邢系
−3.39
2
巴耳末系
H_α
−13.6
1
莱曼系

3.3.2 Spectra of Hydrogenlike Ions

- only 1 electron outside the nucleus with Ze positive charge ions
- Wave number

$$\tilde{\nu} = R_a\left(\frac{1}{n^2} - \frac{1}{n'^2}\right), \qquad R_a = \frac{2\pi^2\{(e)(e)\}^2 m_e}{(4\pi\varepsilon_0)^2 ch^3}\frac{1}{1+\frac{m_e}{M}}$$

$$\boxed{\begin{array}{l}(e)(e) \to (e)(Ze) \\ M \to M_A\end{array}}$$

$$R_a \to \frac{2\pi^2\{(Ze)(e)\}^2 m_e}{(4\pi\varepsilon_0)^2 ch^3}\frac{1}{1+\frac{m_e}{M_A}} = R_A Z^2$$

$$\boxed{\tilde{\nu} = R_A Z^2\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) = R_A\left(\frac{1}{(n/Z)^2} - \frac{1}{(n'/Z)^2}\right)}$$

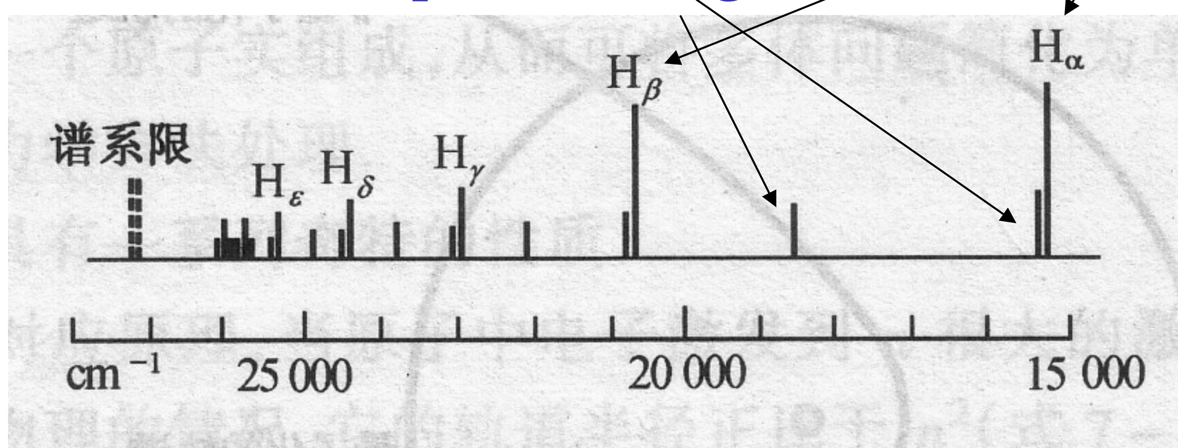
He^+ : $Z = 2$, $M = M_{He+}$, $n = 4$, Pickering series founded

$$\tilde{\nu} = R_{He^+}\left(\frac{1}{(2)^2} - \frac{1}{(n'/2)^2}\right), \quad n' = 5, 6, 7, 8, 9, 10\ldots$$

$$\tilde{\nu} = R_{H}\left(\frac{1}{(2)^2} - \frac{1}{(n')^2}\right), \quad n' = 3, 4, 5, 6, \ldots$$

Balmer

pickering.



3.3.4 The existence of Deuterium

- Deutron -an isotope of H

$${}_1^2H(D) \quad M_D = 2M_H$$

$$\tilde{\nu} = R_D\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \qquad R_D = \frac{2\pi^2 e^4 m_e}{(4\pi\varepsilon_0)^2 ch^3}\frac{1}{1+\dfrac{m_e}{2M_H}}$$

Exp.: H_α **($n' = 3 \rightarrow n = 2$), 6562.79, 6561.00, 1.79**

$$\lambda_H \qquad \lambda_D \qquad \lambda_D - \lambda_H$$

$$\tilde{\nu}_{H_\alpha} = R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R_H$$

$$\lambda_H = \frac{36}{5R_H}, \quad \lambda_D = \frac{36}{5R_D} \qquad \Delta\lambda = \lambda_H - \lambda_D = \frac{36}{5}\frac{R_D - R_H}{R_H R_D} = 1.79$$

Good agreement, proving the existence

3.4 Experimental evidence II :Frank-Hertz experiment

3.4.1 Basic idea

Using e beam to excite atoms:

Excited state

$$e + A = e' + A^*$$

if ***A has discrete energy levels*** (quantized), if only the kinetic energy of ***e*** is higher than the lowest excitation energy of A, can transfer to A, excite it into A^*, causing sudden drop of the kinetic energy of e'。

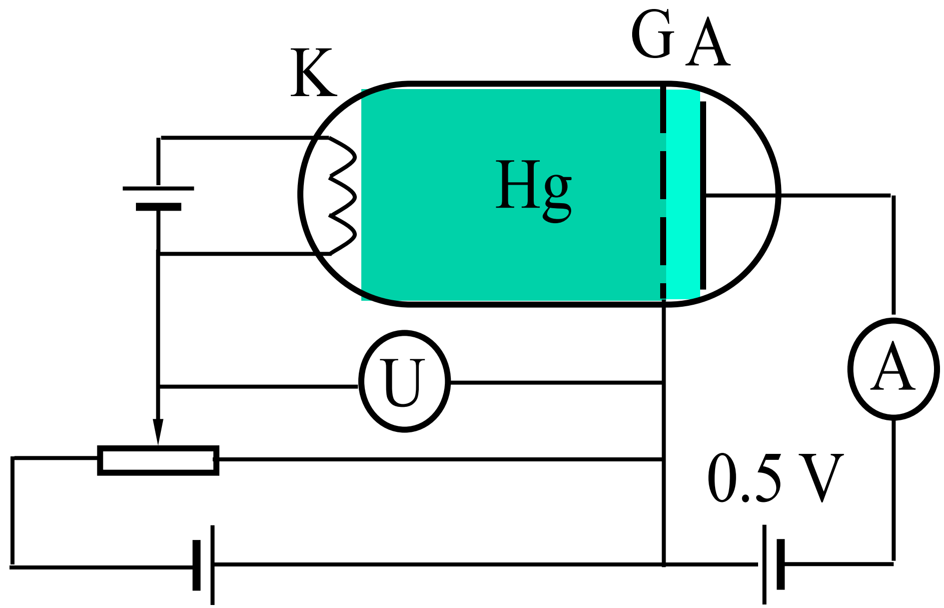
3-4-2 Frank—Herz experiment (1914)

Glass container:gas ; cathode **K:** emitted e;

Between K & positive charged grid G: Btw G and receiver A:

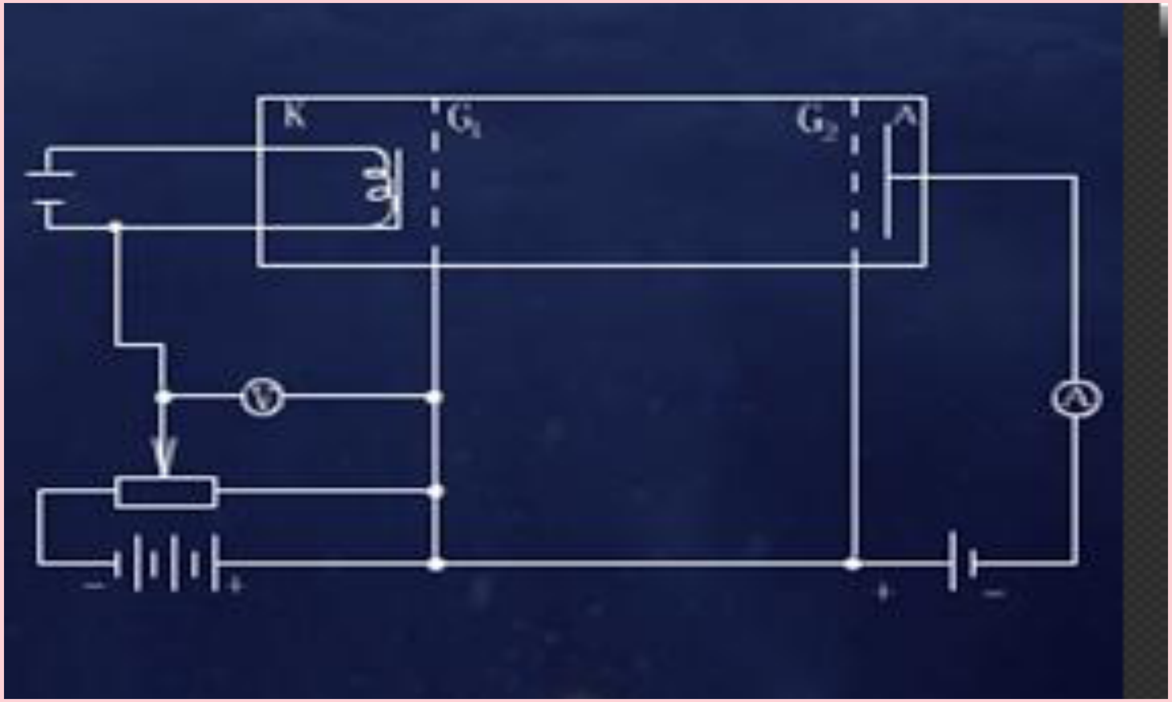
-0.5V negative voltage , Ammeter A

electrons pass through KG enter into GA.→large energy e reach A forming current; small Energy e can not reach A, no current



Electrons in the mercury atoms do not accept just any Energy

3.4.3 Improved Frank—Herz experiment (1920)



1.An electrode plate was added in front K

2 A grid G1 added near K, acceleration in kG1 without collisions

3. G1 and G2 at same electric potential : only collision

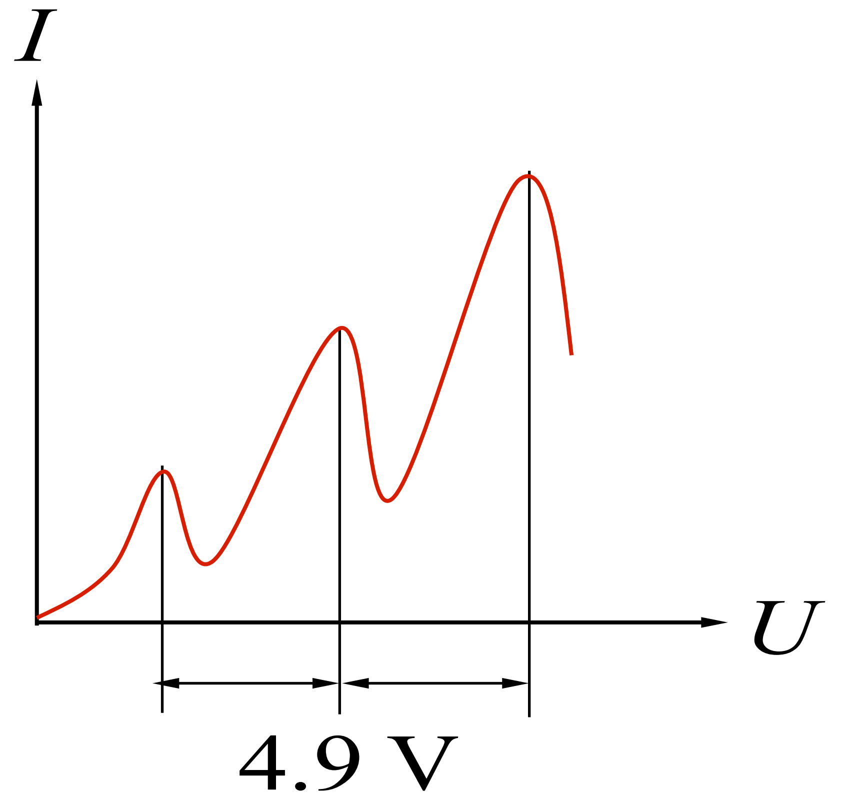
Separate acceleration from collision

Frank-Herz experiment results

Mercury atom vapor

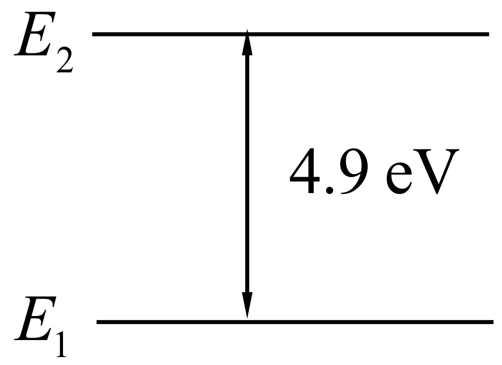
Current ***I versus*** **KG** ***voltage U***

KG U↑ → I↑, ↓, peaks &valley (distance 4.9eV). When KG U= 4.9n, I decreases sharply



Experiment explaination

Existence of
Quantum state E=4.9*e*V



When U < 4.9eV: **elastic** , $U\uparrow \rightarrow$ **Ee**$\uparrow$ $\rightarrow$ $I\uparrow$;

– $U = 4.9$eV: **inelastic** , **excited** E_1 to E_2, **Ee**$\downarrow \rightarrow I\downarrow$

– $4.9\text{eV} < U < 2*4.9\text{eV}$; **Elastic** , $U\uparrow \rightarrow$ **Ee**$\uparrow$ $\rightarrow$ $I\uparrow$

– $U = 2*4.9$eV: **inelastic** , **excited** E_1 to E_2, **Ee**$\downarrow \rightarrow I\downarrow$

3.5 Extension: Bohr-Sommerfeld model

- Relativistic mass

$$m = \frac{m_0}{\sqrt{1-\beta^2}}, \beta \equiv \frac{v}{c}$$

$$v = c \rightarrow \beta = 0 \rightarrow m \approx m_0$$

- Relativitic kinetic energy

$$E_k = (m - m_0)c^2 = m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right)$$

$$v = c \rightarrow \beta = 0 \rightarrow E_k \approx m_0 c^2 \left(1 + \frac{1}{2}\beta^2 - 1 \right) = \frac{1}{2} m_0 v^2$$

Relativistic correction

- Modification of the circular orbits

$$E = E_k - \frac{Ze^2}{4\pi\varepsilon_0 r} \qquad \boxed{r_n = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} n^2,} \qquad \frac{e^2}{4\pi\varepsilon_0 \hbar c} \equiv \alpha$$

$$\frac{Ze^2}{4\pi\varepsilon_0 r_n} = \frac{Z^2 e^2 m e^2}{(4\pi\varepsilon_0)^2 \hbar^2 n^2} = \frac{Z^2}{n^2} \frac{e^4}{(4\pi\varepsilon_0)^2 \hbar^2 c^2} mc^2 = \frac{Z^2}{n^2} \alpha^2 mc^2$$

$$v_n = \alpha c \frac{Z}{n^2}$$

$$E_n = -\frac{1}{2} m_e (\alpha c)^2 \left(\frac{Z}{n}\right)^2 = -T = -\frac{1}{2} m_e v^2$$

$$\boxed{\beta = \frac{v_n}{c} = \frac{\alpha Z}{n^2}}$$

$$E = E_k - \frac{Ze^2}{4\pi\varepsilon_0 r} = (m - m_0)c^2 - mc^2\left(\frac{Z\alpha}{n}\right)^2$$

$$= -m_0c^2 + mc^2\left(1 - \left(\frac{Z\alpha}{n}\right)^2\right) = -m_0c^2 + \frac{m_0c^2}{\sqrt{1-\beta^2}}\left(1-\beta^2\right)$$

$$= m_0c^2\left(\sqrt{1-\beta^2} - 1\right)$$

$$\boxed{\beta = \frac{\alpha Z}{n^2}}$$

$$\boxed{m = \frac{m_0}{\sqrt{1-\beta^2}}}$$

$$E = m_0 c^2 \left(\sqrt{1-\beta^2} - 1 \right)$$

$$\approx m_0 c^2 \left(1 - \frac{1}{2}\beta^2 + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!}\beta^4 - 1 \right) = -m_0 c^2 \left(\frac{1}{2}\beta^2 + \frac{1}{8}\beta^4 \right)$$

$$E \approx -\frac{m_0 c^2}{2}\left(\frac{Z\alpha}{n}\right)^2 - \frac{m_0 c^2}{8}\left(\frac{Z\alpha}{n}\right)^4$$

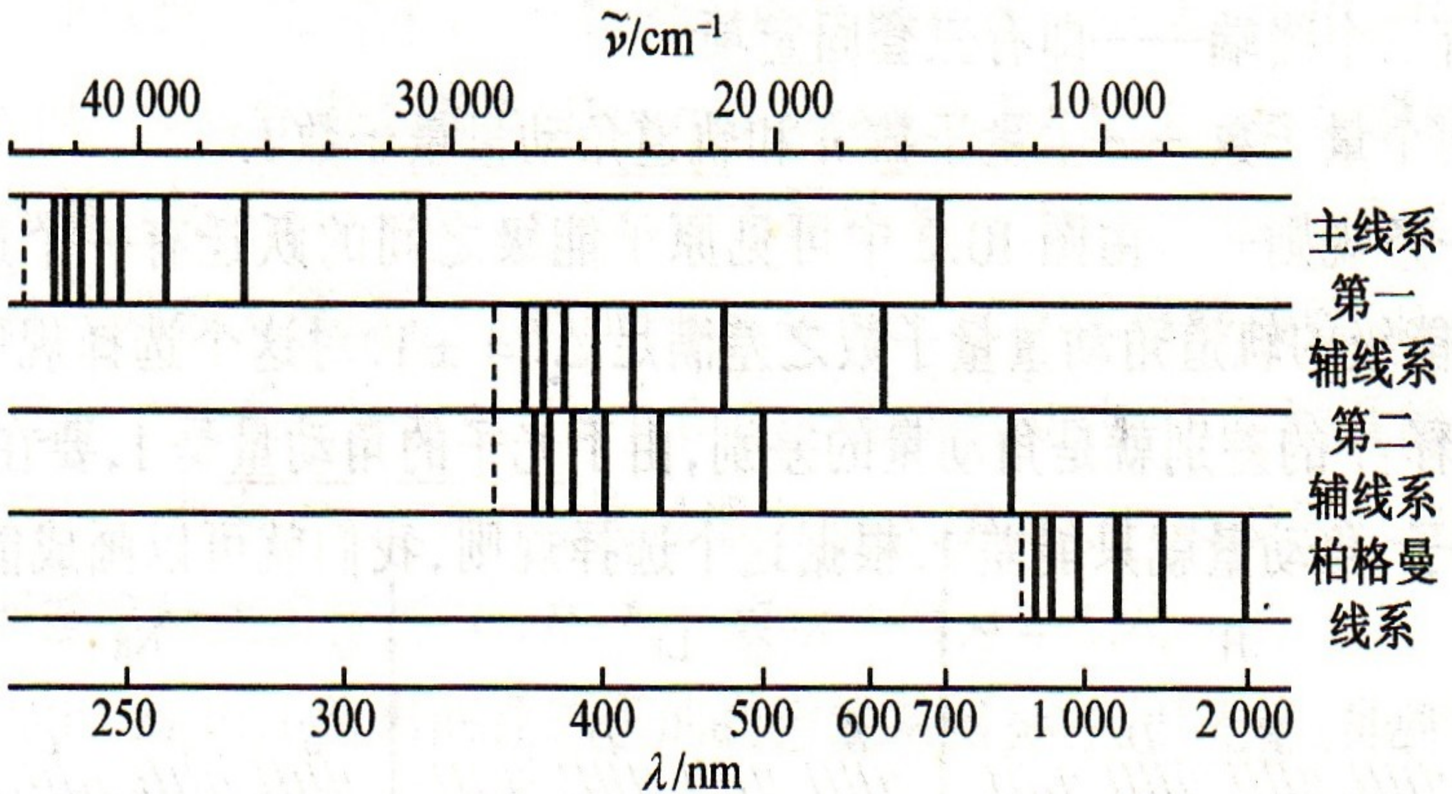
Bohr result

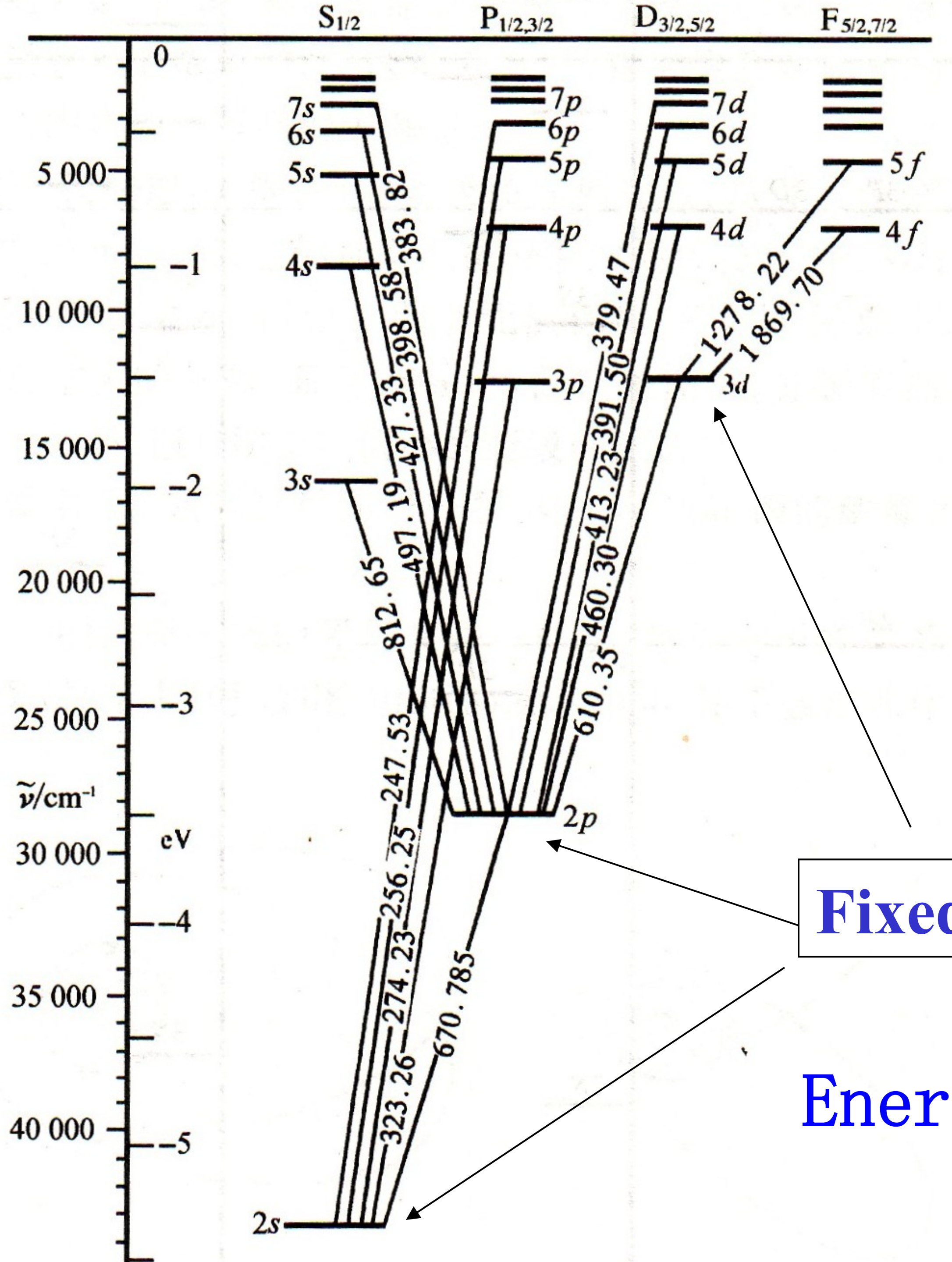
Relativistic correction

Comparison with experiments

- Alkali atoms
  - **The first column : Li, Na, K, Rb, Cs, Fr**
  - **Have one electron outside a closed electron shell.**

Spectral line series of Lithium





Fixed term: **final state**

Energy level of Li

Understanding the spectrum

- Energy levels
  - Combination principle :

Fixing term:
Final state

$$\tilde{\nu} = T(n,l) - T(n',l')$$

Changeable term:
Initial state

$$E_{nl} = -T(n,l)hc$$

- n stands for series; for a finxing ***n***, each ***n'*** ***corresponds to a spectra line***

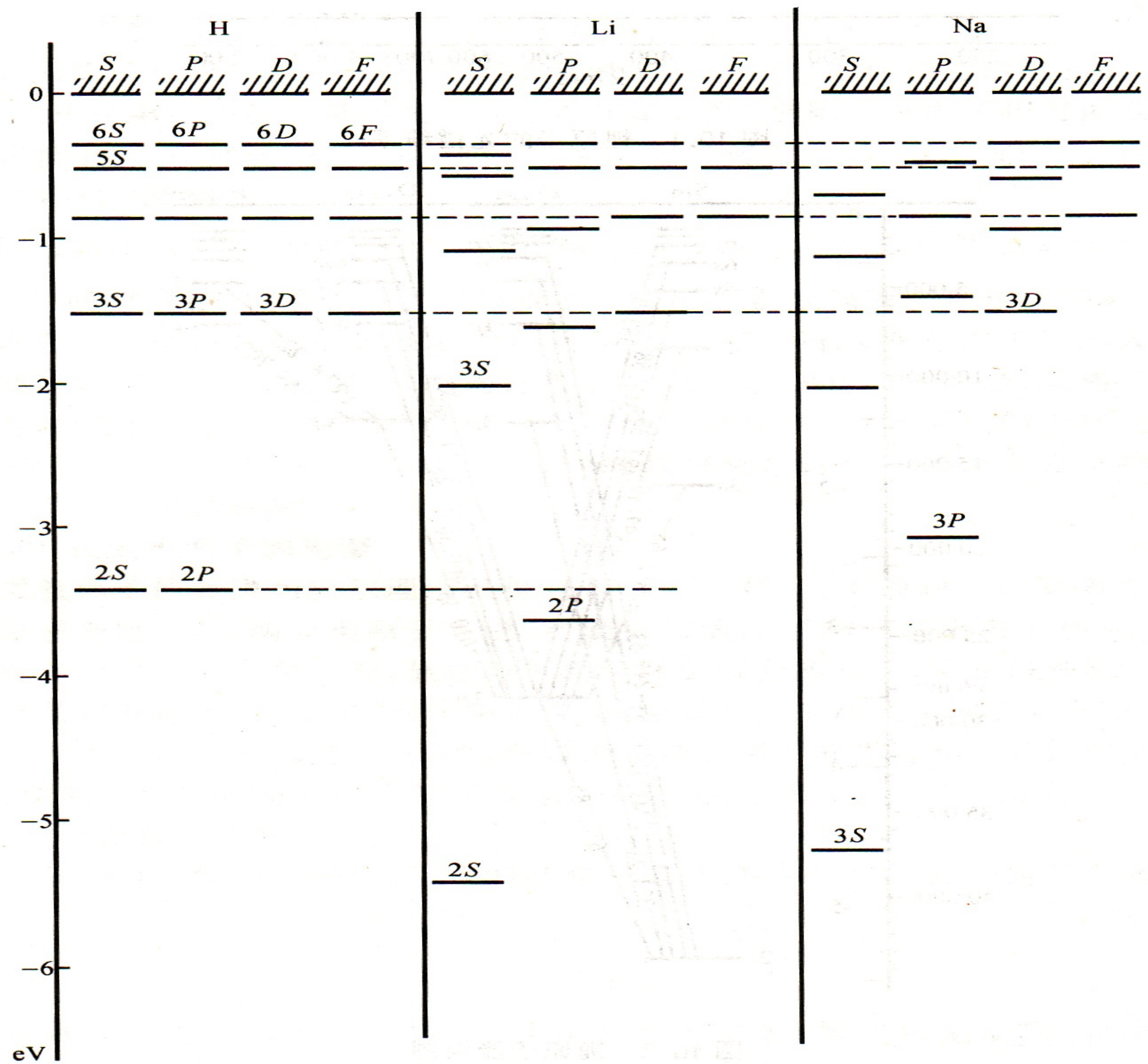
Characters of the energy level

- 4 sets spectral lines： 4 changeable terms
- 3 terminals ： 3 fixed terms
- 2 quantum numbers ： n， l
- 1 rule： selection rule for the transition between 2 atomic energy levels

$$\Delta l = \pm 1$$

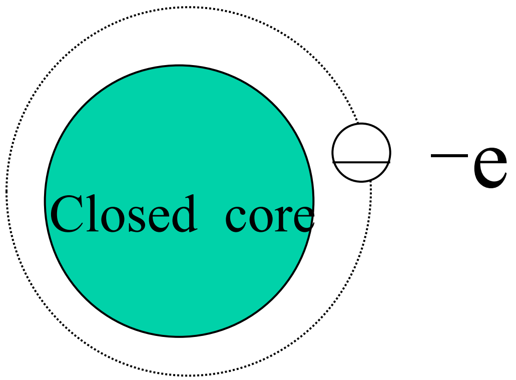
- Energy level classified by l：
- principle：$nP \rightarrow 2S$
- sharp：$nS \rightarrow 2P$（2nd subordinate）
- diffuse：$nD \rightarrow 2P$（primary subordinate）
- fundamental：$nF \rightarrow 3D$（Bergmann）

Comparison of the EL of Hydrogen, Lithium, and sodium



Splitting of the EL of H and Alkali atoms

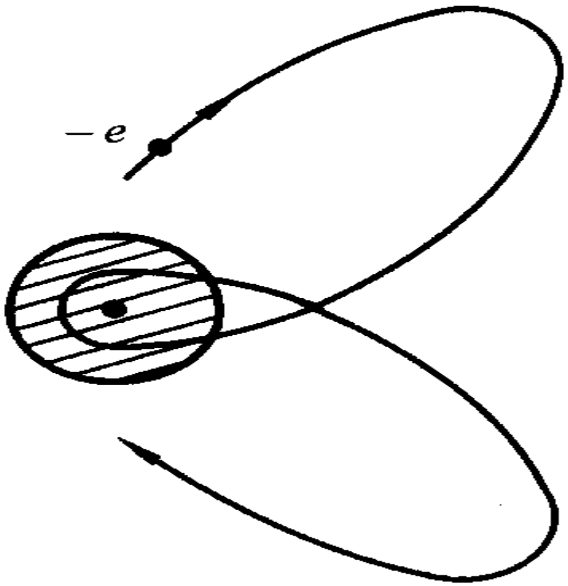
- Alkali atom =
valence e + closed core



for fixing n, l larg→ e is far from the core, **less impact on the core**, similar to H.

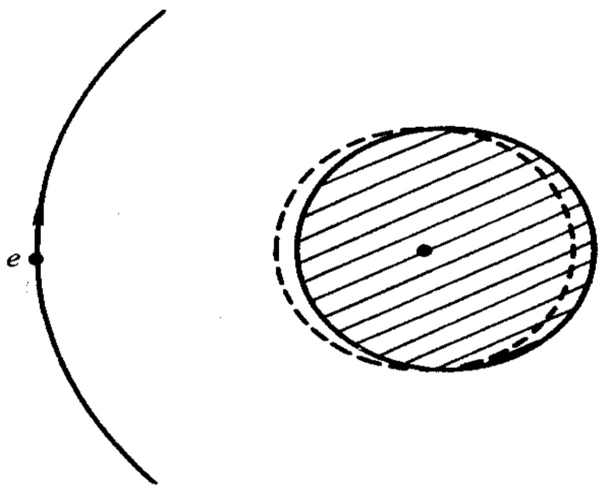
n fixing, l small→ e is close to the core, more impact on the core (penetrating orbit, core polarization), different from H

- Penetrating orbit



l small, e penetrates throug
the core and hence decrease its energy

polarization of the closed atomic core



l small, e close to the core, produce slight relative displacement of the positive and negative charge centers, An electric dipole formed, attracting the electron and decreases its energy

Penetrating robit and polarization explain the splitting between the enery levels of H and Alkali atoms,

3-6 Bohr model's S&D

Success: H spectra of light,

Quantumstates(Frank-Herzexperiment)

Transitions

Difficulties:

(1)intensity of spectrum line, width, polarization

(2) spectrum of multi-e atoms;

(3) selection rules

Need greater revolution!